# Introduction to Natural Language Processing 

Part VIII: NLP using Language Models

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## Learning Objectives

## Concepts

- $n$-gram probability distributions
- Perplexity of language models
- The notion of prompting


## Methods

- Text generation with $n$-gram language models
- Dealing with unknown words in language models
- Different types of smoothing to alleviate model sparsity
- Beam search for improved text generation

Covered tasks

- Free text generation


## Outline of the Course

I. Overview
II. Basics of Linguistics
III. NLP using Rules
IV. NLP using Lexicons
V. Basics of Empirical Methods
VI. NLP using Regular Expressions
VII. NLP using Context-Free Grammars
VIII. NLP using Language Models

- Introduction
- $n$-Gram Language Models
- Advanced Language Modeling
IX. Practical Issues


## Introduction

## Language Models

## Example: Next words

- Given the following sequence of words:


## ChatGPT is based on a neural language

- Which of the following is the most likely next word?

```
that model learning language
```


## Example: Probabilities of word sequences

- Given the following two sequences of words:

```
language models have become a key technique in NLP
NLP models language in key have become a technique
```

- Which of them seems more likely?


## Language Models

n-Gram Language Model

## Language model (LM)

- A language model represents a probability distribution over sequences of tokens, $s=\left(w_{1}, \ldots, w_{k}\right)$, with $k \geq 1$.
- It thus defines the probability $P(s)$ of any token sequence $s$.
- Also, it assigns a probability $P\left(w_{k+1} \mid s\right)$ to any next token $w_{k+1}$ after $s$.


## Where do the probabilities come from?

- $P(s)$ can be approximated by the relative frequency of $s$ in a corpus.
- For longer $s, P(s)$ may be unreliable (or even 0 ) due to data sparsity.
$n$-gram language model
- An $n$-gram LM derives the probability of $s$ from the probability of all token sequences of length $n$ contained in $s$.
- $n \geq 1$ is a predefined hyperparameter of the LM.
- The larger $n$, the more data is needed to get reliable estimations $P(s)$.


## Language Models

Challenges in Language Modeling

## Vanishing probabilities

- In real-world data, the probability of most token sequences $s$ is near 0 , which may lead to vanishing probabilities.
- A way to deal with this problem is to use log probabilites.


## Unknown words and sequences

- Some tokens may never appear in a training corpus.
- Even without unknown tokens, there may always be sequences $s$ that do not appear in training corpus, but appear in other data.
- A technique used to deal with these problems is called smoothing.


## Exactness vs. generalization

- The higher $n$, the more exact the estimated probabilities.
- Sometimes, less context (i.e., a lower $n$ ) may aid generalization.
- Two techniques to deal with this problem are backoff and interpolation.


## Language Models

Applications

## When to use LMs？

－Probabilities of token sequences are essential in any task where tokens have to be inferred from ambiguous input．
－Ambiguity may be due to linguistic variations or due to noise．
－LMs are a key technique in generation，but are also used for analysis．

## Selected applications

－Speech recognition．Disambiguate unclear words based on likelihood．
wreck a nice beach recognize speech
－Spelling／Grammar correction．Find likely errors and suggest alternatives．
I booked one and Tim booked too I booked one and Tim booked two
－Machine translation．Find likely interpretation／order in target language．
爱国人 $\rightarrow$ love country human $\rightarrow$ country loving human

## Language Models

Applications: Free Text Generation

## Free text generation

- Nowadays, the key application of LMs is free text generation.
- Input. An $n$-gram representing the beginning of a text, called the prompt
- Output. The most likely sequence of text following the prompt

Input. Introduction to Natural
Input. What is INLP?
$\rightarrow$ Output. Language Processing is just madness.
$\rightarrow$ Output. Just madness.

How to generate text?

- Stepwise predict the most likely next token (diversity can be enforced).

$$
w_{k}:=\operatorname{argmax}_{w} P\left(w \mid w_{k-(n-1)}, \ldots, w_{k-1}\right)
$$

How to stop generating text?

- The maximum length of the output sequence may be prespecified.
- Also, LMs may learn to generate a special end tag, </s>.


## Outlook: Beyond N-Gram Language Models

Neural language models

- LMs that rely on neural networks to get the probabilities of next tokens
- Main difference: Tokens modeled as real-valued vectors (embeddings)
- This enables generalizing learned dependencies to unseen sequences.

$$
\begin{aligned}
& \text { Training: } \operatorname{argmax}_{w} P(w \mid \text { the people were })=\text { lovely } \\
& \text { Application: } P(\text { lovely } \mid \text { the peepz were })=\text { ? }
\end{aligned}
$$

How are probabilities computed?

- As for an $n$-gram LM, probabilities are derived from a corpus.
- Neural LMs are trained (unsupervised) to predict probabilities.


## Autoregressive text generation

- Stepwise append the most likely next token to the prompt and its previously appended tokens.



## Outlook: Beyond N-Gram Language Models

Large Language Models

Large language model (LLM)

- A neural language model trained on huge amounts of textual data
- Usually based on the transformer architecture


## Transformer

- A neural network architecture for processing input sequences in parallel
- Models each input based on its surrounding inputs, called self-attention
- Examples. GPT-x, BART, T5, ...


Example: ChatGPT https://chat.openai.com

- A dialogue system based on GPT-3.5/GPT-4 that answers reasonably (and often impressively) to nearly any human-written input
- Notice that ChatGPT still has clear limitations, e.g., in terms of factuality.


## n-Gram Language Models

## N-Grams

$n$-gram

- An $n$-gram $s$ is a sequence of $n$ tokens for a fixed $n \geq 1$
- A text with $m \geq n$ tokens consists of $m-n+1$ (overlapping) $n$-grams.
- Example. "The quick brown fox jumps over the lazy dog."

1-grams (unigrams). "The", "quick", "brown", "fox", ..., "dog", "."
2-grams (bigrams). "The quick", "quick brown", ..., "lazy dog", "dog."
3-grams (trigrams). "The quick brown", "quick brown fox", ..., "lazy dog."

## Notation

- $P(w)$. The probability that a variable $X_{i}$ has the value " $w$ ", $P\left(X_{i}=" w\right.$ ")
- $P\left(w_{1}, \ldots, w_{k}\right)$. The joint probability $P\left(X_{1}=" w_{1} ", \ldots, X_{k}=" w_{k}\right.$ " $)$


## Chain rule of probabilities (CRP)

- The joint probability of a sequence of values " $w_{1}$ ",.. , " $w_{k}$ " is defined as:



## N-Grams

Estimating Probabilities

## Problem

- How to determine the probability of "model" in the initial example?


## $P$ (model | ChatGPT is based on a neural language)

## Solution?

- Given a corpus, it can be estimated from frequency counts:
$\frac{\text { \# ChatGPT is based on a neural language model }}{\text { \# ChatGPT is based on a neural language }}$


## Problem

- Even a huge corpus does not allow for good estimates in many cases.
- This is due to language diversity: too many sequences are possible.


## Solution

- Simplify the estimation of probabilities. $\rightarrow n$-gram language model


## N-Grams

Intuition of the $n$-gram Language Model

## Simplification

- Instead of modeling the full history of a token (i.e., all previous tokens), approximate the history by the previous $n-1$ tokens only.
- So, the probability of a token $w_{k}$ given its previous tokens $w_{1}, \ldots, w_{k-1}$ is approximated as follows:

$$
P\left(w_{k} \mid w_{1}, \ldots, w_{k-1}\right) \approx P\left(w_{k} \mid w_{k-(n-1)}, \ldots, w_{k-1}\right)
$$

## Example: Bigrams

- Approximate the probability of token $w_{k}$ given $w_{1}, \ldots, w_{k-1}$ only based on its previous token $w_{k-1}$ :

$$
P\left(w_{k} \mid w_{1}, \ldots, w_{k-1}\right) \approx P\left(w_{k} \mid w_{k-1}\right)
$$

- The conditional probability sought for above is thus simplified to:

$$
P(\text { model } \mid \text { ChatGPT is based on a neural language }) \approx(P(\text { model } \mid \text { language })
$$

## N-Grams

Maximum Likelihood Estimation (MLE)

## Maximum likelihood estimation (MLE)

- In general, the conditional probability of a token $w_{k}$ in a sequence of tokens $s=\left(w_{1}, \ldots, w_{k}\right)$ can be estimated as:

$$
P\left(w_{k} \mid w_{1}, \ldots, w_{k-1}\right) \approx \frac{\#\left(w_{1}, \ldots, w_{k}\right)}{\#\left(w_{1}, \ldots, w_{k-1}\right)}
$$

where \# refers to the count of the sequences in a corpus.

- With the $n$-gram simplification, only $n-1$ previous tokens are modeled:

$$
P\left(w_{k} \mid w_{k-(n-1)}, \ldots, w_{k-1}\right) \approx \frac{\#\left(w_{k-(n-1)}, \ldots, w_{k}\right)}{\#\left(w_{k-(n-1)}, \ldots, w_{k-1}\right)}
$$

- Later, we see how to further adjust the MLE to get better estimates.


## Example: Bigrams

- Only the previous token is modeled:

$$
P\left(w_{k} \mid w_{k-1}\right) \approx \frac{\#\left(w_{k-1}, w_{k}\right)}{\#\left(w_{k-1}\right)}
$$

## N-Gram Language Model

## Language model (LM)

- A probability distribution over a sequence of tokens
- An LM assigns a probability $P\left(w_{1}, \ldots, w_{k}\right)$ to each sequence of tokens $s=\left(w_{1}, \ldots, w_{k}\right)$ for any length $k \geq 1$.
$n$-gram language model
- An LM that approximates the probability of a sequence $s=\left(w_{1}, \ldots, w_{k}\right)$ of $k \geq 1$ tokens for some $n \geq 1$ as:

$$
P\left(w_{1}, \ldots, w_{k}\right)=\prod_{i=1}^{k} P\left(w_{i} \mid w_{1}, \ldots, w_{i-1}\right) \quad \approx \quad \prod_{i=1}^{k} P\left(w_{i} \mid w_{i-(n-1)}, \ldots, w_{i-1}\right)
$$

## Start and end tags

- Start tags. To have a history for the first tokens in $s$ (where $n>i$ ), start tags $\langle\mathrm{s}\rangle$ are prepended to $s$.
$n-1$ start tags must be prepended, in general.
- End tag. $\langle/ \mathrm{s}\rangle$ is appended to $s$ to obtain a true probability distribution.


## N-Gram Language Model

Example: Estimation of Conditional Probabilities
A mini training set with three sentences

```
<s> <s> language models model language </s>
<s> <s> model language as a language model </s>
<s> <s> language models as a model </ s>
```

Selected bigram probabilities (only green tags considered)

$$
\begin{array}{l|c|c}
P(\text { language }|<\mathrm{s}\rangle)=\frac{2}{3} \approx 0.67 & P(\text { model }|<\mathrm{s}\rangle)=\frac{1}{3} \approx 0.33 & P(\mathrm{a}|<\mathrm{s}\rangle)=\frac{0}{3}=0.00 \\
\hline P(\text { models } \mid \text { language })=\frac{2}{5}=0.40 & P(\langle/ \mathrm{s}\rangle \mid \text { language })=\frac{1}{4}=0.25 & P(\mathrm{a} \mid \mathrm{as})=\frac{2}{2}=1.00
\end{array}
$$

Selected trigram probabilities (both blue and green tags considered)

$$
\begin{array}{ll}
P(\text { language }|<\mathrm{s}\rangle\langle\mathrm{s}\rangle)=\frac{2}{3} \approx 0.67 & P(\text { model } \mid\langle\mathrm{s}\rangle\langle\mathrm{s}\rangle)=\frac{1}{3} \approx 0.33 \\
P(\text { models }|<\mathrm{s}\rangle \text { language })=\frac{2}{2}=1.00 & P(\text { as } \mid \text { model language })=\frac{1}{2}=0.5
\end{array}
$$

## N-Gram Language Model

Example: Computation of Sequence Probabilities
A test sentence

$$
s=\langle s\rangle\langle s\rangle \text { model language as a model }\langle/ \mathrm{s}\rangle
$$

Probability computation under bigram LM (only green tags considered)

$$
\begin{aligned}
P_{n=2}(s)= & P(\text { model } \mid\langle s\rangle) \cdot P(\text { language } \mid \text { model }) \cdot P(\text { as } \mid \text { language }) \\
& \cdot P(\mathrm{a} \mid \text { as }) \cdot P(\text { model } \mid \mathrm{a}) \cdot P(</ \mathrm{s}\rangle \mid \text { model }) \\
\approx & 0.33 \cdot 0.5 \cdot 0.2 \cdot 1.0 \cdot 0.5 \cdot 0.67 \approx 0.0111
\end{aligned}
$$

## Probability computation under trigram LM (both blue and green tags considered)

$$
\begin{aligned}
P_{n=3}(s)= & P(\text { model } \mid\langle\mathrm{s}\rangle\langle\mathrm{s}\rangle) \cdot P(\text { language } \mid\langle\mathrm{s}\rangle \text { model }) \\
& \cdot P(\text { as } \mid \text { model language }) \cdot P(\text { a } \mid \text { language as }) \\
& \cdot P(\text { model } \mid \text { as a }) \cdot P(\langle/ \mathrm{s}\rangle \mid \text { a model }) \\
\approx & 0.33 \cdot 1.0 \cdot 0.5 \cdot 1.0 \cdot 0.5 \cdot 1.0=0.0825
\end{aligned}
$$

## N-Gram Language Model

## Practical Issues

## What $n$ to use?

- Bigrams are used in the examples above mainly for simplicity.
- In practice, mostly trigrams, 4 -grams, or 5 -grams are used.
- The higher $n$, the more training data is needed for reliable probabilities.

Besides, notice that LMs may also consider capitalization and non-word tokens.
Log probabilites

- Computations are done in log space to avoid vanishing probabilities.
- Addition in log space is equivalent to multiplication in linear space.
- The actual probabilities can be recovered when needed:

$n$-gram vs. neural LMs
- The $n$-gram LM is the simplest way to map sequences to probabilities.
- Neural LMs extend them but build on the same language modeling idea.


## Evaluation and Application of Language Models

## Evaluation of LMs

- Extrinsic. Measure/Compare impact of LMs within an application.
- Intrinsic. Measure the quality of LMs independent of an application.

Example: Extrinsic evaluation of spelling/grammar correction

```
P(too | booked) vs. P(two | booked) }\quadP(\mathrm{ too | Tim booked) vs. P(two | Tim booked)
```


## Intrinsic evaluation

- Compute all probabilities of an LM on the training set of a corpus.
- Measure the quality the LM on the test set. As usual, a validation set may also be needed during development.

How to measure the quality of an LM intrinsically?

- An LM is better, the higher the probability that it assigns to the test set.
- Rationale: The LM then predicts the test set more accurately.
- The measure used to reflect the probability is called perplexity.


## Evaluation and Application of Language Models

## Perplexity

## Perplexity

- The perplexity PPL of an LM on a test set is the inverse probability of the test set, normalized by the number of tokens.
- If the test set is given as one long sequence, $s=\left(w_{1}, \ldots, w_{m}\right)$, then:

$$
P P L(s)=P\left(w_{1}, \ldots, w_{m}\right)^{-\frac{1}{m}}=\sqrt[m]{\frac{1}{P\left(w_{1}, \ldots, w_{m}\right)}} \stackrel{\mathrm{CRP}}{=} \sqrt[m]{\prod_{i=1}^{m} \frac{1}{P\left(w_{i} \mid w_{1} \ldots, w_{i-1}\right)}}
$$

## Perplexity of bigram LMs

- Under a bigram LM, the perplexity is accordingly computed as follows:

$$
P P L(s)=\sqrt[m]{\prod_{i=1}^{m} \frac{1}{P\left(w_{i} \mid w_{i-1}\right)}}
$$

## Notice

- Each sentence is included in $s$ with start and end tag <s> and </s>.
- The end tags are counted as part of the length $m$ (the start tags not).


## Evaluation and Application of Language Models

Perplexity: Interpretation

## Branching factor (BF)

- The number of next tokens in a language that can follow any token
- Perplexity can be understood as the weighted average branching factor.
- Example. The language of digits, $\Sigma=\{0,1, \ldots, 9\}$

$$
\begin{aligned}
& \text { If } P(w)=0.1 \text { for each } w \in \Sigma \text { in a test set } s \text {, then } B F=10 \text { and } P P L(s)=10 \text {. } \\
& \text { If } P(w)=0.95 \text { for any } w \in \Sigma \text { in a test set } s \text {, then } B F=10 \text { but } P P L(s)<10 \text {. }
\end{aligned}
$$

## Example: Perplexity of $n$-gram models

- Training set. 38 million tokens from Wall Street Journal articles
- Test set. 1.5 milion tokens from other Wall Street Journal articles

$$
\text { Unigram LM: } P P L \approx 962 \quad \text { Bigram LM: } P P L \approx 170 \quad \text { Trigram LM: } P P L \approx 109
$$

## Notice

- Perplexity values are comparable only for LMs with same vocabulary.
- Better (so, lower) perplexity does not imply more extrinsic effectiveness.


## Evaluation and Application of Language Models

Sequence Sampling

## Sampling of sequences

- The probabilities of an LM encode knowledge from the training set.
- To see this, sequences $s$ can be sampled based on their likelihood $P(s)$.


## Unigram sampling

- Decompose the probability space $[0,1]$ into intervals, each reflecting the probability of one unigram from the LM vocabulary.

- Choose a random point in the space, and write the associated unigram.
- Repeat this process until $</ s>$ is written.


## Bigram sampling

- Same technique, starting by sampling random $w_{1}=w$ from $P\left(w_{1} \mid<s>\right)$.
- Repeat process for $P\left(w_{2} \mid w\right)$ and so forth, until $\langle/ \mathrm{s}\rangle$ is written.


## Evaluation and Application of Language Models

Text Generation using Sequence Sampling

## Example: Sampling from Shakespeare's works (900k words, 29 k unique words)

1-grams:
To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have

Why dost stand forth thy canopy, forsooth; he is
2-grams: this palpable hit the King Henry. Live king. Follow.

Fly, and will rid me these news of price. Therefore the
3-grams: sadness of parting, as they say, 'tis done.

Hill he late speaks; or! a more to leg less first you enter

What means, sir. I confess she? then all sorts, he is trim, captain.

This shall forbid it should be branded, if

$$
0
$$ renown made it empty.

4-grams: King Henry. What! I will go seek the traitor Gloucester.

It cannot be but so. Exeunt some of the watch. A great banquet serv'd in;

## Observations

- As $n$ is increased, $n$-gram LMs improve in generating coherent text.
- Under a 4-gram LM, some sequences are just copies of Shakespeare. The reason is data sparsity: $7 \cdot 10^{17}$ possible 4 -grams, but less than 900 k examples.

Advanced Language Modeling

## Advanced Language Modeling

## Sparsity

- $n$-grams frequent in a training set may get reliable probability estimates.
- But even huge training sets will not contain all possible $n$-grams.


## Example: Wall Street Journal Treebank

- Counts of trigrams starting with "denied the":

```
# denied the allegation = 5 ... rumors = 1 ... speculation = 2 ... report =1
```

- Probabilities of other trigrams starting with "denied the":

$$
P(\text { denied the offer })=0 \quad P(\text { denied the loan })=0
$$

Why are zero probabilities problematic?

- The probability of any unknown token (sequence) is underestimated.
- If any test set probability is 0 , the probability of the entire test set is 0 . What is the perplexity in this case?
- No next token can be predicted for any unknown token or sequence.


## Advanced Language Modeling

## Unknown Tokens

## Out-of-vocabulary (OOV) tokens

- OOV tokens are those that appear in a test set but not in a training set.
- They are unknown to an LM built on the training set.
- Common examples. Slang words, misspellings, URLs, rare words, ...


## Solution

- Replace all unknown tokens in a test set by a special tag, <UNK>.
- As for any token, estimate the probability of <UNK> on the training set.
- Two common ways to obtain <UNK> training instances exist.


## Alternative 1: Closed vocabulary

1. Choose a fixed vocabulary of known tokens in advance.
2. Convert any other (OOV) token to <UNK>.

Alternative 2: Frequency pruning

1. Choose a minimum absolute or relative frequency threshold, $\tau$.
2. Convert any token with training frequency $<\tau$ to <UNK>.

## Smoothing

## Unknown sequences

- Even if all tokens in a sequence $s$ are known, $s$ as a whole might have never appeared in a training set.
- Techniques to avoid that $P(s)=0$ in such cases are called smoothing.


## General idea of smoothing (aka discounting)

- Reduce the probability mass of known sequences.
- Distribute gained mass over unknown sequences.



## Main types of smoothing

- Laplace smoothing and Add-k smoothing
- Backoff, simple interpolation, and conditional interpolation
- Absolute discounting and Kneser-Ney smoothing
- Stupid backoff


## Smoothing

Laplace Smoothing

## Laplace smoothing (aka add-1 smoothing)

- Add 1 to the count of all $n$-gram counts before estimating probabilites.

So, an unseen $n$-gram gets a count of 1 , one with count 100 has $101, \ldots$

## Unigram MLE under Laplace smoothing

- Given a training set with $m$ tokens, the unsmoothed unigram probability estimate of a token $w$ is:

$$
P(w)=\frac{\# w}{m}
$$

- If the vobulary size is $v$, then the MLE of $w$ is modified to:

$$
P_{\text {Laplace }}(w):=\frac{\# w+1}{m+v}
$$

## Notice

- Laplace smoothing is not used in practice, due to issues shown below.
- Rather, it shows the key smoothing idea and may serve as a baseline.


## Smoothing

Laplace Smoothing: Example
Modified bigram counts and unigram counts for the mini training set

|  | language | model | models | as | a | $\langle/ \mathbf{s}\rangle$ |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| <s> | $2+1=3$ | $\ldots$ | 2 | 1 | 1 | 1 | 1 |
| language | $0+1=1$ | $\cdots$ | 2 | 3 | 2 | 1 | 2 |
| model | $\ldots$ | 3 |  | 1 | 1 | 1 | 1 |
| models | 1 |  | 2 | 1 | 2 | 1 | 1 |
| as | 1 |  | 1 | 1 | 1 | 3 | 1 |
| a | 2 |  | 2 | 1 | 1 | 1 | 1 |


| \# Unigram |
| ---: |
| 3 |
| 5 |
| 4 |
| 2 |
| 2 |
| 2 |

## Bigram probability estimation

- Under Laplace smoothing, the bigram probabilities are estimated as:

$$
P_{\text {Laplace }}\left(w_{i} \mid w_{i-1}\right):=\frac{\#\left(w_{i-1}, w_{i}\right)+1}{\# w_{i-1}+v}
$$

- Selected probabilities, given the vocabulary of size $v=6$ :

$$
P_{\text {Laplace }}(\text { language }|<\mathrm{s}\rangle)=\frac{2+1}{3+6} \approx 0.33 \quad P_{\text {Laplace }}(\text { models } \mid \text { model })=\frac{0+1}{4+6}=0.10
$$

- Some probabilities are strongly reduced, as $P$ (language $\mid\langle s\rangle)$ here. Before, $P($ language $\mid\langle s\rangle)=0.67$, as seen above.


## Smoothing

Add- $k$ Smoothing

## Problem with Laplace smoothing

- Adding one to all counts may strongly change the probabilities.
- Too much probability mass is moved to all the (former) zero counts.
- A relaxation is to do add-k smoothing instead.

This does not solve the problem, though. Further refinements follow below.

## Add- $k$ smoothing

- Add only a fractional count $k$ to the count of all $n$-grams, $0<k<1$.
- $k$ is a hyperparameter that can be optimized on a validation set.

Typical values might be $k=0.5, k=0.05$, or $k=0.01$.

Bigram probability estimation

- Under add- $k$ smoothing, the bigram probabilities are estimated as:

$$
P_{\text {Add }-k}\left(w_{i} \mid w_{i-1}\right):=\frac{\#\left(w_{i-1}, w_{i}\right)+k}{\# w_{i-1}+k \cdot v}
$$

## Backoff and Interpolation

## Less context for better generalization

- If an $n$-gram probability cannot be computed, it can be approximated by probabilities of subsequences.
- Example. $P\left(w_{i} \mid w_{i-1}\right)$ and/or $P\left(w_{i}\right)$ may be used for $P\left(w_{i} \mid w_{i-2}, w_{i-1}\right)$.
- Two techniques to limit context this way are backoff and interpolation.


## Backoff

- Reduce $n$ by 1 , if an $n$-gram probability $P\left(w_{i} \mid w_{i-(n-1)}, \ldots, w_{i-1}\right)=0$.
- Repeat until $P\left(w_{i} \mid w_{i-(n-1)}, \ldots, w_{i-1}\right)>0$ (latest at $n=1$, if <UNK> used).
- To maintain a probability distribution, discount higher-order $n$-grams.


## Katz backoff

- Discount probabilities $P^{*}$ for known $n$-grams.
- Use function $\lambda$ to assign probabilities to lower-order $n$-grams of others. $P^{*}$ and $\lambda$ are estimated using Good Turing smoothing (beyond the scope here).
$P_{\mathrm{KB}}\left(w_{i} \mid w_{i-(n-1)}, \ldots, w_{i-1}\right):=\left\{\begin{array}{l}P^{*}\left(w_{i} \mid w_{i-(n-1)}, \ldots, w_{i-1}\right) \text { if } \#\left(w_{i-(n-1)}, \ldots, w_{i}\right)>0 \\ \lambda\left(w_{i-(n-1)}, \ldots, w_{i-1}\right) \cdot P_{\mathrm{KB}}\left(w_{i} \mid w_{i-(n-2)}, \ldots, w_{i-1}\right) \text { else }\end{array}\right.$


## Backoff and Interpolation

## Interpolation

## Interpolation

- Always mix weighted probability estimates from all $n$-gram estimators.
- Weights are usually chosen such that they maximize the likelihood of a validation set (i.e., minimizing perplexity).


## Simple interpolation

- Combine different order $n$-gram probabilities via linear interpolation, using weights $\lambda_{j}$ with $\sum_{j} \lambda_{j}=1$
- Example. Unigrams, bigrams, and trigrams:

$$
P_{\mathrm{SI}}\left(w_{i} \mid w_{i-2}, w_{i-1}\right):=\lambda_{1} \cdot P\left(w_{i}\right)+\lambda_{2} \cdot P\left(w_{i} \mid w_{i-1}\right)+\lambda_{3} \cdot P\left(w_{i} \mid w_{i-2}, w_{i-1}\right)
$$

## Conditional interpolation

- Condition each weight $\lambda_{j}$ on the given context:

$$
\begin{aligned}
P_{\mathrm{Cl}}\left(w_{i} \mid w_{i-2}, w_{i-1}\right):= & \lambda_{1}\left(w_{i-2}, w_{i-1}\right) \cdot P\left(w_{i}\right)+\lambda_{2}\left(w_{i-2}, w_{i-1}\right) \cdot P\left(w_{i} \mid w_{i-1}\right) \\
& +\lambda_{3}\left(w_{i-2}, w_{i-1}\right) \cdot P\left(w_{i} \mid w_{i-2}, w_{i-1}\right)
\end{aligned}
$$

## Absolute Discounting

## Discounting

- Smoothing discounts frequent sequences, to save probability for unknown sequences.
- Question: How much discounting is best?

Idea of absolute discounting

- Compare training set count to mean count on some validation set.

Table: Bigram counts in a 22M words news corpus.

- Choose fixed discount value $d$ on this basis.

| \# Bigrams |  |  |
| ---: | ---: | ---: |
| Train. | Valid. | $\Delta$ |
| 1 | 0.45 | 0.55 |
| 2 | 1.25 | 0.75 |
| 3 | 2.24 | 0.76 |
| 4 | 3.23 | 0.77 |
| 5 | 4.21 | 0.79 |
| 6 | 5.23 | 0.77 |
| 7 | 6.21 | 0.79 |
| 8 | 7.21 | 0.79 |
| 9 | 8.26 | 0.74 |

(Interpolated) Absolute discounting

- Subtract a fixed absolute discount $d$ from each count.
- Distribute gained probability mass weighted over lower-order $n$-grams:

$$
\begin{aligned}
P_{\mathrm{AD}}\left(w_{i} \mid w_{i-(n-1)}, \ldots, w_{i-1}\right):= & \frac{\#\left(w_{i-(n-1)}, \ldots, w_{i}\right)-d}{\#\left(w_{i-(n-1)}, \ldots, w_{i-1}\right)} \\
& +\lambda\left(w_{i-(n-1)}, \ldots, w_{i-1}\right) \cdot P\left(w_{i} \mid w_{i-(n-2)}, \ldots, w_{i-1}\right)
\end{aligned}
$$

## Smoothing

Kneser-Ney Smoothing: Intuition

## Kneser-Ney Smoothing in a nutshell

- Absolute discounting with a refined handling of lower-order distributions
- One of the best $n$-gram smoothing methods proposed so far


## Unigram intuition of refinement

## ChatGPT is based on a neural language

- A default unigram model will assign "york" a higher probability than "model", since "york" is more frequent in general.
- But "model" appears in many contexts, "york" mostly in "new york" only.


## Refined lower-order $n$-gram handling

- Define probability of a sequence $s=\left(w_{1}, \ldots, w_{k}\right)$ from its likelihood to appear in novel contexts.
- Derive estimate from number of unigrams continued by $s$ in a corpus:

$$
\#\left\{w_{0}: \#\left(w_{0}, \ldots, w_{k}\right)>0\right\}
$$

## Smoothing

Kneser-Ney Smoothing

## (Interpolated) Kneser-Ney Smoothing

- Use count \# for highest order and continuation count for lower orders.
- Discount counts by $d$ as in absolute discounting.
- Recursively distribute probability mass, normalized by a constant $\lambda$.
$\lambda$ is the normalized discount, multiplied by how often it is applied (see below).

$$
\begin{aligned}
P_{\mathrm{KN}}\left(w_{i} \mid w_{i-(n-1)}, \ldots, w_{i-1}\right):= & \frac{\max \left(c_{\mathrm{KN}}\left(w_{i-(n-1)}, \ldots, w_{i}\right)-d, 0\right)}{c_{\mathrm{KN}}\left(w_{i-(n-1)}, \ldots, w_{i-1}\right)} \\
& +\lambda\left(w_{i-(n-1)}, \ldots, w_{i-1}\right) \cdot P_{\mathrm{KN}}\left(w_{i} \mid w_{i-(n-2)}, \ldots, w_{i-1}\right)
\end{aligned}
$$

where

$$
c_{K N}\left(w_{1}, \ldots, w_{k}\right):= \begin{cases}\#\left(w_{1}, \ldots, w_{k}\right) & \text { for highest-order } n \text {-grams } \\ \#\left\{w_{0}: \#\left(w_{0}, w_{1}, \ldots, w_{k}\right)>0\right\} & \text { for lower-order } n \text {-grams }\end{cases}
$$

and

$$
\lambda\left(w_{i-(n-1)}, \ldots, w_{i-1}\right):=\frac{d}{\#\left(w_{i-(n-1)}, \ldots, w_{i}\right)} \cdot \#\left\{w_{i}: \#\left(w_{i-(n-1)}, \ldots, w_{i}\right)>0\right\}
$$

## Large N-Gram Language Models

## Large $n$-gram language models

- The larger the training set, the more reliable the estimated probabilities
- By employing web-scale text corpora, extremely large LMs can be built.


## Example $n$-gram corpora

- Google Web NGrams. 1 trillion English word $n$-grams, $1 \leq n \leq 5$, all with 40+ occurrences
- Google Books NGrams. 800 billion token $n$-grams in eight languages

| 4-grams | Count |
| :--- | ---: |
| serving as the independent | 794 |
| serving as the index | 223 |
| serving as the indicator | 120 |
| serving as the incubator | 99 |
| serving as the incoming | 92 |
| $\ldots$ | ... |

## Efficiency challenges

- The number of sequences and resulting $n$-gram probabilities explodes.
- Technical space optimizations may be necessary, such as hashing.
- To reduce time and space needs, less frequent $n$-grams can be pruned.


## Large N-Gram Language Models

Stupid Backoff

## Smoothing under large LMs

- It is possible to realize Kneser-Ney smoothing at web scale.
- Alternatively, however, the scale enables the resort to a much simpler method called stupid backoff.


## Stupid backoff

- Do not discount higher-order probabilities, i.e., drop the requirement to have a true probability distribution.
- If a higher-order $n$-gram is unknown, approximate its probability from a lower-order $n$-gram, weighted by a constant weight $\lambda$.
$\lambda=0.4$ has been found to work well in experiments.
$S\left(w_{i} \mid w_{i-(n-1)}, \ldots, w_{i-1}\right):= \begin{cases}\#\left(w_{i-(n-1)}, \ldots, w_{i}\right) & \text { if } \#\left(w_{i-(n-1)}, \ldots, w_{i-1}\right)>0 \\ \# \cdot\left(w_{i-(n-1)}, \ldots, w_{i-1}\right) & \\ \lambda \cdot S\left(w_{i} \mid w_{i-(n-2)}, \ldots, w_{i-1}\right) & \text { otherwise }\end{cases}$


## Evaluation and Application of Language Models

Netspeak: Writing Support based on $n$-Grams

## Netspeak One word leads to another.

> English

German

| professor ? artificial intelligence |  |  |
| :--- | ---: | ---: |
| professor of artificial intelligence | 953 | $66 \%$ |
| professor, artificial intelligence | 440 | $30 \%$ |
| professor in artificial intelligence | 59 | $4.1 \%$ |
|  |  |  |

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https://netspeak.org

## Evaluation and Application of Language Models

Improving Results in Text Generation

## Beam search

- A simple heuristic search method often used to find optimal sequences
- Instead of extending only the most likely sequence $s^{*}$, extend all $\beta>1$ most likely sequences to finally generate best.
- Rationale: Another sequence $s \neq s^{*}$ may turn out better later.



## Diversification using randomization

- A simple way to generate more diverse text is to randomize each step.
- Instead of writing the most likely token $w_{k+1}$, write any of the top $l \geq 1$.


## Prompting

- The quality of the output of an LM always depends on the prompt.
- This is why prompt engineering is currently a hot topic in academia and industry (but beyond the scope of this course).

Conclusion

## Conclusion

## NLP using language models (LMs)

- LMs are probability distributions over token sequences
- Nowadays, one of the most central NLP techniques
- Used particularly for free text generation
$n$-gram language models
- Estimation of probabilities from $n$ tokens only
- The higher $n$, the more training data is needed

- The quality of an LM can be quantified as perplexity


Advanced language modeling

- Smoothing enables dealing with unknown sequences
- Backoff/Interpolation reduce context for generalization
- Different techniques to improve outputs exist



## References

## Much content and multiple examples taken from

- Jurafsky and Martin (2021). Daniel Jurafsky and James H. Martin. Speech and Language Processing: An Introduction to Natural Language Processing, Speech Recognition, and Computational Linguistics. Draft or 3rd edition, December 29, 2021. https://web.stanford.edu/ jurafsky/slp3/

